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Lesson 19: Four Interesting Transformations of Functions

Student Outcomes

* Students examine that a horizontal scaling with scale factor of the graph of corresponds to changing the equation from to .

Lesson Notes

In this lesson, students study the effect a horizontal scaling by scale factor has on the graph of an equation . For example, if , a horizontal scaling by will horizontally shrink any geometric figure in the Cartesian plane, including figures that are graphs of functions. The horizontal scaling of a graph corresponds to changing the equation from to . For values of scale factor where , the graph of is a horizontal stretch of the graph of by a factor of .

In this lesson, students may employ MP.3 when they make conjectures about the effect of , MP.8 when they use repeated reasoning to determine the effect of , and MP.6 when the communicate the effect to others using careful language.

Classwork

Students explore the horizontal scaling of the graph of when the equation changes from to for . In this case, students see the graph of is a horizontal “shrink” by . In Example 1, the scale factor for is , or , or .

Example 1 (8 minutes)

Example 1

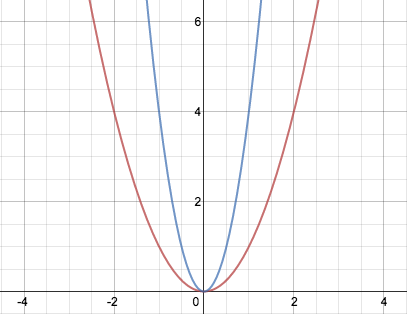
Let and , where can be any real number.

* 1. Write the formula for in terms of (i.e., without using notation):

1. Complete the table of values for these functions.

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1. **Graph both equations: and .**



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See the discussion below for an explanation of the steps and arrows.

1. How does the graph ofrelate to the graph of ?

**The corresponding -value of is half of the corresponding -value of when , the points of the graph of are ½ the distance to the -axis as the corresponding points of the graph of , which makes the graph of g appear to “shrink horizontally.”**

1. How are the values of related to the values of ?

For equal outputs of and , the input of only has to be half as big as the input of .

**Discussion (5 minutes)**

* A horizontal scaling of a graph with scale factor will “shrink” the original graph horizontally by and correspond to the graph of the equation or , i.e., the horizontal scaling of the graph of with scale factor is the graph of the equation .
* In Example 1, what process could be used to find the value of for any given number , using only the graph of (not the formula for )?
  + *Step 1: Find on the -axis.*
  + *Step 2: Multiply by to find the number on the -axis.*
  + *Step 3: Find the value of at .*
  + *Step 4: Move parallel to the -axis from the point found in Step 3 until directly over/under/on . That point is . [These steps are numbered and illustrated in the graph above for .]*

Lightly erase the graph of (already drawn from part (c)), and then go through the steps above to redraw it, picking out a few points to help students see that *only* the -values are changing between corresponding points on the graph of and the graph of . If you erased the graph lightly enough so that the “ghost” of the image is still there, students will see that you are redrawing the graph of over the original graph. Following the steps will give students a sense of how the points of the graph of are only “shrinking” in the -values, not the -values.   
  
Many students might confuse a horizontal scaling with other types of transformations like dilations. In fact, a dilation with scale factor of the graph of in this example produces the exact same *image* as a horizontal scaling by , but the correspondence between the points is *different*. Your goal in Grade 9 is to have students develop a “rigid” notion of what a vertical scaling means so that it can be profitably compared to dilation in Grades 10 and 11.

* Consider a function , and a transformation of that function , such that how do the domain and range of relate to the domain and range of ?
  + *The range of both functions will be the same, but the domains may change.*
* What might the graph of look like?
* What might the graph of look like if it were graphed on the same Cartesian plane as the graphs of and ?

Let students go up to the board and draw their conjectures on the plane.

**Discussion (5 minutes)**

Students explore the horizontal scaling of the graph of when the equation changes from to for . In this case, students see that the graph of is horizontally “stretched” by a factor of . In Example 2, the scale factor for is , or .

Example 2 (8 minutes)

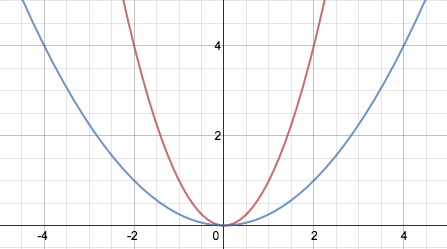
Example 2

Let and , where can be any real number.

1. Rewrite the formula for in terms of (i.e., without using notation):
2. **Complete the table of values for these functions.**

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1. **Graph both equations: and .**



1. How does the graph of relate to the graph of?

**Since the corresponding -value of is twice the corresponding -value of when , the points of the graph of are 2 times the distance to the -axis as the corresponding points of the graph of , which makes the graph of appear to “stretch horizontally.”**

1. How are the values of related to the values of ?

**To get equal outputs of each function, the input of has to be twice the input of .**

A horizontal scale of a graph with scale factor will “stretch” the original graph horizontally by 2 and correspond to the graph of the equation , i.e., the horizontal scale of the graph of with scale factor is once again the graph of the equation . Follow the steps given in Discussion 1 to show students how to find the value on the Cartesian plane *using only the graph of*  (not the formula for ). Emphasize that only the -values are being scaled. When comparing to , the range of both functions will be the same, but the domains may change. Ask students what the graph of might look like after a horizontal scale with scale factor . Let them draw their conjecture on the graph on the board. Then ask them what the equation of the resulting graph is.

Exercise 1 (6 minutes)

Have students discuss the following exercise in pairs. Discuss the answer as a class.

Exercise 1

Complete the table of values for the given functions.

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* 1. Label each of the graphs with the appropriate functions from the table.



* 1. Describe the transformation that takes the graph of to the graph of .

**The graph of is a horizontal scale with scale factor of the graph of**

* 1. Consider . What does negating the input do to the graph of ?

**The graph of is a reflection over the -axis of the graph of .**

* 1. Write the formula of an exponential function whose graph would be a horizontal stretch relative to the graph of .

**Answers will vary. Example: .**

Example 3 (6 minutes)

**Example 3**

* 1. **Look at the graph of for the function in Example 1 again. Would we see a difference in the graph of if was used as the scale factor instead of ? If so, describe the difference. If not, explain why not.**

**There would be no difference. The function involves squaring the value within the parentheses, so the graph of and the graph of both will be the same set as the graph of , but both correspond to different transformations: The first is a horizontal scaling with scale factor , and the second is a horizontal scaling with scale factor and a reflection across the -axis.**

* 1. **A reflection across the -axis takes the graph of for the function back to itself. Such a transformation is called a *reflection symmetry.* What is the equation for the graph of the reflection symmetry of the graph of ?**

**.**

Tell students that if a function satisfies the equation for every number in the domain of , it is called an *even function*. A consequence of an even function is that its graph is symmetrical with respect to the -axis. Furthermore, the graph of is symmetrical across the -axis. A reflection across the -axis does not change the graph.

* 1. Deriving the answer to the following question is fairly sophisticated; do only if you have time: In Lessons 17 and 18, we used the function to examine the graphical effects of transformations of a function. Here in Lesson 19, we use the function to examine the graphical effects of transformations of a function. Based on the observations you made while graphing, why would using be a better option than using the function?

**Not all of the effects of multiplying the input of a function are as visible with an absolute function as it is with a quadratic function. For example, the graph of is the same as . Therefore, it is easier to see the effect of multiplying a value to the input of a function by using a quadratic function than it is by using the absolute value function.**

Closing (2 minutes)

Discuss how the horizontal scaling by a scale factor of of the graph of a function corresponds to changing the equation of the graph from to . Investigate the four cases of :

Exit Ticket (5 minutes)

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Lesson 19: Four Interesting Transformations of Functions

Exit Ticket



Let and , where can be any real number. The graphs above are of , , and .

1. Label each graph with the appropriate equation.
2. Describe the transformation that takes the graph of to the graph of . Use coordinates of each to illustrate an example of the correspondence.
3. Describe the transformation that takes the graph of to the graph of . Use coordinates to illustrate an example of the correspondence.

Exit Ticket Sample Solutions

Let and , where can be any real number. The graphs above are of , , and .

1. Label each graph with the appropriate equation.



See graph.

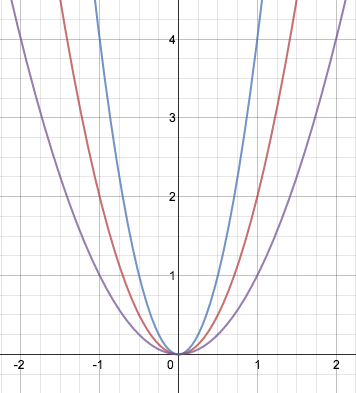
1. Describe the transformation that takes the graph of to the graph of . Use coordinates of each to illustrate an example of the correspondence.

**The graph of is a horizontal shrink of the graph of with scale factor . The corresponding -value of is one-third of the corresponding -value of when . This can be illustrated with the coordinate on and the coordinate on .**

1. Describe the transformation that takes the graph of to the graph of . Use coordinates to illustrate an example of the correspondence.

**The graph of is a horizontal stretch of the graph of with scale factor 3. The corresponding -value of is three times the corresponding -value of when . This can be illustrated with the coordinate on and the coordinate on .**

Problem Set Sample Solutions



Let and , where can be any real number. The graphs above are of the functions and .

1. Label each graph with the appropriate equation.

See graph.

1. Describe the transformation that takes the graph of to the graph of . Use coordinates to illustrate an example of the correspondence.

**The graph of is a vertical stretch of the graph of by scale factor 2; for a given -value, the value of is twice as much as the value of .**

**OR:  
 The graph of is a horizontal shrink of the graph of by scale factor ; it takes times the input for as compared to to yield the same output.**

1. Describe the transformation that takes the graph of to the graph of . Use coordinates to illustrate an example of the correspondence.

**The graph of is a horizontal shrink of the graph of by a scale factor of ½; It takes the input for as compared to to yield the same output.**

**OR:**

**The graph of is a vertical stretch of the graph of by scale factor 4; for a given -value, the value of is four times as much as the value of .**